

# Entrance Exam

Mathematics

Jun 26, 2020

This exam contains 8 groups of problems. Please write your solutions in the space provided. Use the back if you need more room. Include all the details of the solution process, partial credit may be awarded. Results without details or without reasoning are not acceptable. The total of available points is 80.

Please note this is a “closed book” exam. The use of notes of any kind is not permitted in this examination. Please do not use your cell phone or any other electronic devices. Sign your name on all of your papers at the bottom right.

Your Name (capitalize, please):


Problem #	Score won
1:	
2:	
3:	
4:	
5:	
6:	
7:	
8:	
Total:	

**Problem 1.**

(4+4+4 points)

Simplify the following expressions:

$$(a^2 - b^2 - c^2) : \frac{a + b - c}{a + b + c};$$

$$49^{1-\log_7 2} + 5^{-\log_5 4};$$

$$\left[ \frac{x^{-3}y^2}{x^{-4}y} \right]^{-2}.$$

**Problem 2.**

(4+4 points)

Solve the following equations:

$$\frac{ax - b}{a + b} + \frac{bx + a}{a - b} = \frac{a^2 + b^2}{a^2 - b^2};$$

$$2^{\cos^2 x} = \sin x.$$

**Problem 3.**

(2+4+4 points)

Set up a quadratic equation, whose roots are  $\frac{1}{10-\sqrt{72}}$  and  $\frac{1}{10+6\sqrt{2}}$ .

The roots  $x_1$  and  $x_2$  of the quadratic equation  $x^2 + px + 12 = 0$  possess the following property:  $x_1 - x_2 = 1$ . Find the coefficient  $p$ .

Solve the system of equations:

$$\begin{aligned}x^3 + y^3 &= 7 \\xy(x + y) &= -2\end{aligned}$$

**Problem 4.**

(5+5 points)

Assume that  $a \neq 0$ . Solve equation

$$1 + a + a^2 + \dots + a^{x-1} + a^x = (1 + a) (1 + a^2) (1 + a^4) (1 + a^8).$$

Solve the equation

$$\left(\frac{3}{7}\right)^{3x-7} = \left(\frac{7}{3}\right)^{7x-3}.$$

**Problem 5.**

(9 points)

A tailor shop has an order for 810 suits, another shop has to make 900 suits in the same period of a time. The first shop has completed its task 3 days before the target day and the second 6 days ahead of time. How many suits does each shop produce per day, if the second shop makes 4 suits per day more than the first.

**Problem 6.**

(6+6 points)

Compute the area of a trapezoid whose parallel sides are equal to 16 and 44, and the nonparallel ones are 17 and 25.

Two sides of a triangle are 10 and 16. The area of the triangle is 48. Find the length of the third side!

**Problem 7.**

(4+5 points)

Solve the following equation

$$1 - \sin 5x = \left( \cos \frac{3}{2}x - \sin \frac{3}{2}x \right)^2 .$$

Solve the inequality

$$\sqrt{1 - \sin 2x} \geq \sin 2x - 1.$$



**Problem 8.**

(10 points)

Find all rational zeros of the polynomial

$$p(x) = x^4 - \frac{7}{2}x^3 - x^2 + \frac{13}{2}x + 3.$$