5. Fejezet – Megoldások

5.1 alfejezet

1. A tipikus célfüggvény szintvonal a $3x_1+c_2x_2=z$. Ennek $-3/c_2$ a meredeksége. Amennyiben ez <-2, a C pont az optimális. Tehát, ha $-3/c_2<-2$, azaz $c_2<1.5$, a jelenlegi bázis többé már nem optimális. Hasonlóan, ha a célfüggvény szintvonalának meredeksége >-1 az A pont lesz optimális. Tehát, ha $-3/c_2>-1$, azaz $c_2>3$, a jelenlegi bázis többé már nem optimális. Kapjuk, hogy $1.5\leq c_2\leq 3$ esetén marad a jelenlegi bázis optimális.

Ha $c_2 = 2.5$, akkor $x_1 = 20$, $x_2 = 60$, viszont $z = 3(20) + 2.5(60) = \$210$.

2. Currently Number of Available Carpentry Hours = b_2 = 80. If we reduce the number of available carpentry hours we see that when the carpentry constraint moves past the point (40, 20) the carpentry and finishing hours constraints will be binding at a point where $x_1>40$. In this situation $b_2<40 + 20 = 60$. Thus for $b_2<60$ the current basis is no longer optimal. If we increase the number of available carpentry hours we see that when the carpentry constraint moves past (0, 100) the carpentry and finishing hours constraints will both be binding at a point where $x_1<0$. In this situation $b_2>100$. Thus if $b_2>100$ the current basis is no longer optimal. Thus the current basis remains optimal for $60\leq b_2\leq 100$. If $60\leq b_2\leq 100$, the number of soldiers and trains produced will change.

3. If b_3, the demand for soldiers, is increased then the current basis remains feasible and therefore optimal. If, however, $b_3<20$ then the point where the finishing and carpentry constraints are binding is no longer feasible (it has $s_3<0$). Thus for $b_3>20$ the current basis remains optimal. For $b_3>20$ the point where the carpentry and finishing constraints are still binding remains at (20, 60) so producing 20 soldiers and 60 trains remains optimal.

4a. If isocost line is flatter than HIM constraint, point C is optimal. If isocost line is steeper than HIW, point B is optimal. Thus current basis remains optimal if $-7/2\leq -c_1/100\leq -1/6$ or $50/3\leq c_1\leq 350$.

4b. Current basis remains optimal if $-7/2\leq -50/c_2\leq -1/6$ or $100/7\leq c_2\leq 300$.

4c. If we decrease HIW requirement optimal solution moves towards D = (0, 2). We lose feasibility if HIW requirement is 2(2) = 4. Increase HIW requirement and optimal solution moves towards C = (12, 0). We lose feasibility when HIW requirement = 7(12) = 84. Thus current basis remains optimal for 4,000,000≤HIW Requirement≤84,000,000. If HIW = 28 +Δ, the optimal solution is where $7x_1 + 2x_2 = 28 + \Delta$ and $2x_1 + 12x_2 = 24$. This yields $x_1 = 3.6 + .15\Delta$ and $x_2 = 1.4 -.025\Delta$.

4d. If we decrease HIM requirement optimal solution moves towards A = (4, 0). We lose feasibility if HIM requirement is 2(4) = 8. Increase HIM requirement and optimal solution moves towards B = (0, 14). We lose feasibility when HIM requirement = 14(12) = 168. Thus current basis remains optimal for 8,000,000≤HIW Requirement≤168,000,000. If HIM = 28 +Δ, the optimal solution is where $7x_1 + 2x_2 = 28$ and $2x_1 + 12x_2 = 24 +\Delta$. This yields $x_1 = 3.6 - \Delta/40$ and $x_2 = 1.4 + 7\Delta/80$.

4e. If HIW requirement is 28 +Δ, $z = 50x_1 + 100x_2 = 320 + 5\Delta$ so HIW shadow price is –5 (thousand). If HIM requirement is 24 + Δ, $z = 50x_1 + 100x_2 = 320 + 7.5\Delta$, so HIM shadow price is –7.5.

4f. New z-value = 320 - (-2) (-5) = $310,000

5a. Let c1 = profit from type 1 radio. Slope of isoprofit line is –c1/2. Current basis is still optimal if of isoprofit line is steeper than L1 constraint and flatter than L2 constraint or–2≤–c1/2≤-.5 or 1≤c1≤4 or $23 = 22 + 1≤Type 1 price≤22 + 4 = $26.

5b. Current basis remains optimal for –2≤–3/c2≤-.5 or 1.5≤c2≤6 or $21.50≤Type 2 price≤26.

5c. Optimal solution still occurs where both constraints are binding or $x_1 + 2x_2 = 30$ and $2x_1 + x_2 = 50$. This yields $x_1 = 70/3$, $x_2 = 10/3$, and z = 230/3. Since solution is feasible, it is optimal.

5d. Optimal solution occurs where $x_1 + 2x_2 = 40$ and $2x_1 + x_2 = 60$. This yields $x_1 = 80/3$, $x_2 = 20/3$, z = 280/3.

5e. If 40 + Δ laborer 1 hour are available optimal solution is where $x_1 + 2x_2 = 40 + \Delta$ and $2x_1 + x_2 = 50$. This yields $x_1 = 20 - \Delta/3$, $x_2 = 10 + 2\Delta/3$, $z = 80 + \Delta/3$. Thus laborer 1 shadow price = 1/3.
If 50 + Δ laborer 2 hours are available optimal solution is where $x_1 + 2x_2 = 40$ and $2x_1 + x_2 = 50 + \Delta$.

This yields $x_1 = 20 + \Delta/3$, $x_2 = 10 - \Delta/3$, $z = 80 + 4\Delta/3$. Thus laborer 2 shadow price = 4/3.